Informative Advertising in Vertical Relationships: When Revealing Unfavorable Information is Profitable*

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Abstract

Many advertisements, for example on TV, contain descriptions about quality aspects of a good. This is especially true for advertisements of manufacturing firms which (also) indirectly sell through retailers. A priori, it is not clear how valuable this information is for consumers.

In contrast to much of the existing literature, interpreting the target of informative advertising to be horizontal, that is attracting consumers from competing firms, this paper offers a different explanation.

A simple model of vertical relations, where consumers differ in their valuation of quality, is presented. It is shown that a monopolistic manufacturer can use advertising to transmit private information about the quality of his good to consumers, thereby influencing the incentives of a (monopolistic) retailer and reducing the severity of the well-known double marginalization problem. The cases of experience goods and search goods are distinguished and it is shown that information transmission is especially effective if consumers find it difficult to determine the quality of the good in question.

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1 Introduction

In many situations, consumers are uncertain about aspects of a good they plan to purchase. This uncertainty about the characteristics of the good often means that consumers are not able to ascertain their valuations of the good in advance.

The connection between uncertainty about aspects of the good and consumers' valuation is especially obvious if the the quality of the good is the aspect the consumer is in doubt of.

What may make the assessment of a good's quality even harder for consumers is a situation in which the good is not sold by the manufacturer directly but instead through some kind of vertical chain. In such a setting every agent has its own incentives which the consumer has to take into consideration when trying to assess information about the quality of the good in question.

At the same time it seems prevalent that manufacturers send large amounts of information to consumers, for example through TV advertising where characteristics are explicitly stated or through using reviews carried out by third parties to convince consumers of the superiority of their good.

The problem in those situations, as mentioned above, is that the manufacturer clearly follows his own goals when sending out such information and there is good reason for the consumer to question the credibility of the information provided.

Building on this motivation, this paper will model a situation where a monopolistic manufacturer produces a good with a quality that is unknown to consumers and sold by a monopolistic retailer. All consumers prefer goods of higher quality, but they differ in how much they prefer high quality goods over low quality ones. Since the quality is unknown to consumers, it is impossible for them to determine their exact valuation for the good they consider purchasing. The question then will be if the manufacturer can credibly reveal part of his private information to consumers via cheap-talk and why he would choose to do so.

It is easy to see that this can not happen in a model where the manufacturer directly sells his goods to consumers. In such a model the manufacturer gets the full retail price and since consumers' utility is decreasing in the charged price, when they buy the good, the interests of consumers' and the manufacturer are completely opposed. All consumers' utilities are increasing in the quality and they all possess the same information, so that if any signal the manufacturer directs to consumers would influence their willingness to pay (WTP), the manufacturer would always send the signal which maximizes consumers' WTP. In other words, because of the opposing interests and the costless nature of cheap-talk signals, there is no way to align the incentives of consumers and the manufacturer in a way that opens the possibility of informative signaling as in Crawford and Sobel (1982).

The situation is quite different if we are dealing with a vertical chain setting where the manufacturer sells his good through a retailer. For such a setup it will be shown that the manufacturer can indeed reveal part of his private information and he will choose to do so when he is given the opportunity. The interesting question here is why he would reveal that his good is of relatively low quality to consumers, even though their valuations are increasing in quality, especially since he has no incentive to do so in the direct selling setup. It will be shown that by generating two opposing effects, the vertical chain setup is crucial in enabling the manufacturer to credibly reveal parts of his private information.

While the effect in place in the direct selling setup, namely that signaling higher qualities increases consumers' WTP is still present in a vertical chain model, a second effect emerges. Since the manufacturer does not directly sell his good to consumers but does so only through a retailer, he has to make sure that the retailer has no incentive to deviate from the proposed strategy. Intuitively, both the retailer and the manufacturer have to decide between setting high prices and selling to few consumers or setting low prices and selling to many consumers. In the present model, the gap between consumers' valuations increases for higher qualities, which means that it will get harder for the manufacturer to make the retailer stick to selling to many consumers at a relatively low price the higher consumers expect the quality to be. Put differently, by signaling a low quality good to consumers, the manufacturer decreases the incentives of the retailer to deviate from a strategy that prescribes serving many customers at a low price, in fact, the manufacturer does not only decrease the incentives but also limits the deviation possibilities of the retailer.

In the terminology of Nelson (1970), the cases of experience goods and search goods will be distinguished. In the first case, consumers can only observe the good's quality after consuming it, whereas in the second case they can learn the quality at some costs, which are often interpreted as costs for visiting the point of sale.

For both kinds of goods the introduced possibility of transmitting information to consumers will be valuable for the manufacturer. Yet, the extent to which this is true, differs. The effect of the transmitted information is greater the harder it is for consumers to find out about the quality from other channels of information. In the case of experience goods, information transmission will in general be feasible and useful to consumers, whereas the informativeness of the sent information with search goods depends on how costly it is for consumers to learn about the quality on their own. Intuitively, information provided by the manufacturer will be less useful when a visit to the retailer where the quality of the good is observed gets cheaper.

The paper proceeds as follows, the next section gives an overview of the related literature, Section 3 introduces the model which is solved for different information settings in Section 4. Section 5 extends the model to search goods and Section 6 concludes. The proofs are relegated to the appendix.

2 Related Literature

The model in this paper combines two popular topics in the industrial organization literature.

On the one hand, the manufacturer in this paper transmits information revealing part of his private information about the quality of the good to the consumers. Information transmission in the industrial organization literature is often seen to take the form of advertising. The topic of informative advertising is quite common in the industrial organization literature (for a survey see Bagwell, 2007). In contrast to most of the papers discussing issues of advertising where it is costly, here the manufacturer uses cheap-talk like in Crawford and Sobel (1982) as the advertising device.

On the other hand, this paper deals with the issue of double-marginalization first noticed by Cournot (1863). This issue arises whenever a monopolistic retailer sells a good to a retailer who then sells it to the consumer because both firms charge a markup on the prices they set. Here the manufacturer will use his private information to influence the markup charged by the retailer, influencing the double marginalization effect.

The remainder of this section first presents related work dealing with the questions on how information is transmitted in different market settings and then describes the existing work that deals with such questions in a manufacturer-retailer setup.

2.1 Information Transmission

In general, there is a distinction between the topics of 'Quality Disclosure' and 'Quality Signaling', the distinction being the fact that in the case of disclosure, information can not be misrepresented (but withheld) and in signaling situations, misrepresenting is possible.

For both cases there is a great amount of work in the economic literature (for a model unifying both strands see Daughety and Reinganum, 2007).

The disclosure literature generally argues that firms prefer to disclose their quality since if there were a pooling of firms with different qualities, the highest quality firms benefit from disclosing information, thereby increasing their profits (an early paper in this context is Milgrom, 1981). Given that the highest type would disclose its qualities, consumers reason that the quality must be lower if they do not observe that the quality is disclosed. This in turn leads the second highest quality firm to also disclose and so on. This result is commonly referred to as 'unraveling' and if disclosure is costless, firms of all quality types will disclose in equilibrium.

The literature about signaling usually concentrates on the price and/or on (costly) advertising as the instruments used to inform consumers about the quality. The idea that (seemingly uninformative) advertising may improve consumers' information about a good's quality was first put forward by Nelson (1970) and later formalized by Milgrom and Roberts (1986) who also added the possibility that the prices inform consumers. They find that in a model of repeated purchases of an experience good, a pricing scheme together with the optimal amount of advertising can signal quality to consumers. For more recent work see for example Guo and Zhao (2009) and Levin et al. (2009).

Apart from these articles there is some work on the effect of different forms or intensities of competition on the disclosure decision (Board, 2009 and Guo, 2009) or the ability to use the price as a signal of quality (Janssen and Roy, 2010, Adriani and Deidda, 2011). Although the studies mentioned come to somewhat different results, the general message is that the ability to inform consumers about a product's quality is higher, the higher the market power of the informing agent.

Besides using prices and advertising to signal quality other instruments found in the literature are specialization (Kalra and Li, 2008) and most prominently certification of another agent (for a survey see Dranove and Jin, 2010).

2.2 Vertical Chains

Surprisingly there is rather little literature on the issue of quality disclosure and signaling in a vertical trade setting, the exceptions being Chu and Chu (1994) and Guo (2009). Chu and Chu model a situation where a manufacturer of a high-quality product 'rents' the reputation of a retailer and thereby convinces the consumers that his product is of high quality. Guo models how a manufacturer would choose to disclose quality, either disclosing directly or leaving the decision to the retailer.

A relatively new area of work deals with the reasoning for so-called retail price recommendations (RPR). Work in this area emerged only recently and can be divided into three branches, all dealing with models of vertical trade and information transmission in such models.

Buehler and Gärtner (2013) model a situation where one retailer repeatedly sells the good of one manufacturer to consumers and the manufacturer has private information about his costs of production. Since the information about costs is necessary to maximize the joint profits of the manufacturer and the retailer, they show that this information can be communicated from the manufacturer to the retailer using RPRs.

The following articles consider the target of the information sent by the manufacturer to be the consumers, just as in the paper at hand. They can be divided by the degree of rationality imposed on consumers. Puppe and Rosenkranz (2011) and Fabrizi et al. (2010) employ models of behavioral economics and also consider a vertical trade setting, but in their model consumers have reference dependent preferences. They then assume that RPRs, sent from a manufacturer, are used by the consumers as their reference point and they show that this can increase the profits of the manufacturer.

Lubensky (2011) builds a model where a manufacturer possesses better information about the state of demand than rational searching consumers, who do not observe the price prior to visiting a retailer. He then shows that the manufacturer can send signals about the state of demand to consumers, thereby influencing their search decision and increasing his profits.

While the result in Lubensky's article, that a manufacturer can communicate private information to consumers, is similar to the results that will be presented later, there are some fundamental differences. First of all, the models differ since he assumes one manufacturer and a continuum of retailers whereas a vertical chain of two monopolists is assumed in this paper. However, the crucial difference lies in the kind of information that the manufacturer is better informed of. In Lubensky's model, the manufacturer possesses better information about demand than consumers do. This information is not directly relevant to consumers, but it gets valuable only through its influence on the retail price. As a consequence of this, his model does not allow to evaluate the different implications that experience and search goods have. In the present paper, consumers are uninformed about the quality of the good so that this distinction can easily be made.

3 Model Setup

The following describes the model setup, starting with the two firms who are producing and selling the product and then describing the preferences of the consumers.

The model consists of a manufacturer (M) who produces a good of random quality q with the quality being distributed according to a distribution function $F(q)$ with full support on $Q = [0, \overline{q}]$. M then sells the good to a retailer (R) (who observes the realized quality before) at the wholesale price w and the retailer in turn sells the good to consumers at the retail price p. The retailer doesn't face any costs for selling the good.

The manufacturer's unit costs of production depend on the quality and are given by $c(q)$ with $c(0) = 0 < k - (m^2 - 1)E(q) < c(\overline{q}) < k$, where m denotes the total number of consumers and k gives the consumers' valuation for a good with quality $q = 0$ (see below). Higher quality products are assumed to be more costly to produce, so that $c'(q) > 0$. Besides normalizing the costs of the lowest quality to zero, the assumption guarantees that separation is feasible ($k - (m^2 - 1)E(q) < c(\overline{q})$) as will be shown later and that the good can profitably be sold irrespectively of the realized quality ($c(\overline{q}) < k$).

The manufacturer can directly communicate with consumers, as he is able to send some costless (cheap-talk) signal $\tilde{q}$ to consumers. While this signal need not be restricted and potentially might be anything, it is useful to interpret the signal $\tilde{q}$ as a level of quality.

Assuming random quality is motivated by the idea that even big companies are not able to fully control the produced quality, especially so if they depend on parts they have to buy from other companies.

A prevalent example can be seen in the so-called "Antennagate" of Apple's iPhone 4. There the design of the Antenna led to problems with certain holding positions. Since Apple gave out free cases to complaining consumers, later reached a settlement that alternatively offered consumers 15$[1] and finally changed the antenna design for the model's successor, it seems reasonable to assume that the problems were unforeseen and we can talk about random quality here. In the manner of this example we could also interpret the random quality assumption as describing the

[1] see the official site of the settlement at: https://www.iphone4settlement.com/ (last accessed November 25th, 2016)

uncertainty that remains with the manufacturer when he starts to produce the good. Technically the assumption is needed to make information transmission profitable for the manufacturer. If he were to choose the quality which is produced (and without introducing some other random element), there would generally be one optimal quality that M will prefer, thus taking away the need to signal information. Phrased differently, for information transmission to be valuable, there must be something that M can possibly inform consumers about. Without any randomness, the equilibrium would generally depend on parameters only so that consumers would be able to infer all equilibrium values and information transmission would be unnecessary and of no use.

The good in consideration is what is commonly called an 'Experience Good' (Nelson, 1970), which in this case means that consumers are not aware of the good's quality until after they consumed it.

There are two groups of consumers, the 'low types' ($i = \ell$) and the 'high types' ($i = h$). The groups are of size n_ℓ and n_h respectively, all consumers have unit-demand. The two groups differ in how they value quality improvements of the good. Without loss of generality, n_ℓ will be set to 1, so that the total number of consumers equals $m = n_h + 1$.

The (ex-post) monetary valuations of the consumers for a good of quality q take the form

$$v_i(q) = k + \theta_i q \qquad i \in \{\ell, h\}$$

where k, the "base-line" utility is a positive constant, and the type θ_i gives consumer i's sensitivity to quality improvements, with $1 \geq \theta_h > \theta_\ell \geq 0$.

Thus, when consumer i buys the good, his (ex-post) utility is given by

$$U_i(q) = k + \theta_i q - p \qquad i \in \{\ell, h\}$$

If any consumer decides not to buy the good, his utility is assumed to be zero. Since consumers do not observe the quality prior to consuming the good they have to form beliefs about it using the information they observe, namely retail prices and quality signals. A belief β in the usual sense is a probability distribution over types of the manufacturer (quality realizations), that is, $\beta : (p, \tilde{q}) \to \Delta Q$. In the model at hand, consumers optimally buy the good whenever their expected valuation for it exceeds the retail price, i.e. only the expected quality for a given belief $\mu(p, \tilde{q}) := E_{\beta(p,\tilde{q})}(q)$ is relevant for their decision. This expectation will be referred to when making statements about consumers' beliefs. With the assumed valuations and because the quality level is at least equal to zero, it is optimal for any consumer to buy the good whenever $p < k$, independent of the expected quality. Therefore beliefs for prices lower than k can be chosen arbitrarily.

In contrast to the quality, consumers are able to observe the price before making the purchase decision. Assuming that prices are directly observable simplifies the model and can be seen as a shortcut for a model in which consumers can learn the price only at some positive cost but where the retailer is able to commit to charging some retail price, for example by advertising it.

Because the retailer cannot distinguish consumers, he can potentially sell the good to all consumers at a lower price or to the high-type consumers only at a higher price. His profits, which will be denoted by π_R, then are $1+n_h$ and n_h times the retail minus the wholesale price, respectively. Similarly the manufacturer's profits (denoted by π_M) are $1+n_h$ (n_h) times the wholesale price minus the unit cost $c(q)$, when selling to all (the high-type consumers only).

In this model a strategy of the manufacturer gives a pair of wholesale price and quality signal depending on the realized quality. The retailer's strategy is the retail price potentially depending on the realized quality, quality signal and the wholesale price. A consumer's strategy simply is the decision to buy or not to buy the good, given the price and quality signal he observed.

Figure 1 summarizes the timing and actions of all agents.

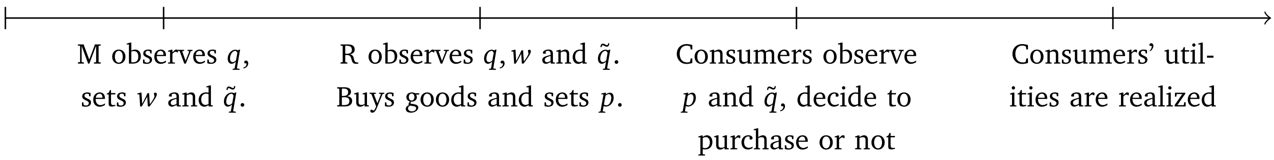

Figure 1: Model Timing

In the following, we will concentrate on (weak) perfect Bayesian equilibria (PBE) in pure strategies, which in this setting requires that

Requirement 1. *Firms maximize their profit given the other firm's and consumers' strategies. Consumers act optimally given the strategies of the firms and their beliefs μ.*

Requirement 2. *Consumers' beliefs are derived from the firms' strategies and Bayes' rule whenever possible.*

Since this solution concept allows for off-equilibrium path beliefs that are not 'credible' (Sadanand and Sadanand, 1995), an equilibrium refinement based on perfect sequentiality (Grossman and Perry, 1986) as formulated in Gertner et al. (1988) will be imposed. In particular, this refinement rules out equilibria where prices are not monotone in the quality. Roughly, the refinement requires that there is no deviation which is profitable for some set of types (of manufacturers or retailers), given consumers attribute this deviation to exactly those types.

Formally, an interpretation $I(p,\tilde{q})$ of a price and quality signal pair not on the equilibrium path, is a set of qualities to which the consumer attributes this deviation, i.e. $I:(p,\tilde{q})\to Q$. Using this interpretation, consumers form their belief given the interpretation, $\mu(p,\tilde{q}) = E[q|q\in I(p,\tilde{q})]$ and act optimally given this belief.

An interpretation is consistent if the types to which consumers attribute the deviation indeed prefer to deviate. In the vertical chain we have to distinguish two possible deviations that can be observed by consumers and thus need to be interpreted, the retailer can deviate by setting a retail price not on the equilibrium path or the manufacturer can send an off-equilibrium path quality signal.

Given an equilibrium with prices w^* and p^*,signal $\tilde{q}^*$ and beliefs $\mu(p^*, \tilde{q}^*)$ an interpretation of the retailers' deviation to p', $I(p', \tilde{q}^*) = T \subseteq Q$ is consistent if

$$\begin{aligned}
\pi_R(p^*, w^*, \mu(p^*)) &\leq \pi_R(p', w^*, I(p', \tilde{q}^*)) \quad \forall t \in T \\
\pi_R(p^*, w^*, \mu(p^*)) &> \pi_R(p', w^*, I(p', \tilde{q}^*)) \quad \forall t \notin T
\end{aligned}$$

Since the manufacturers profits depend directly only on the wholesale price he earns, a deviation in the quality signal that is observed by consumers must be accompanied by a deviation in the wholesale pricing scheme if it is to be profitable for M. This different wholesale price as well as the consumers' interpretation of a different quality signal also make it necessary for the retailer to adjust his strategy. Thus, given a deviation of M from the equilibrium signal $\tilde{q}^*$ to $\tilde{q}'$ together with changing the wholesale price from w^* to w', the interpretation $I(p^{BR}, \tilde{q}') = T \subseteq Q$ where p^{BR} is the retailer's best response to M's changed wholesale price and given consumers' interpretation, is consistent if:

$$\begin{aligned}
\pi_M(w^*, \tilde{q}^*, p^*, \mu(p^*, \tilde{q}^*)) &\leq \pi_M(w', \tilde{q}', p^{BR}, I(p^{BR}, \tilde{q}')) \quad \forall t \in T \\
\pi_M(w^*, \tilde{q}^*, p^*, \mu(p^*, \tilde{q}^*)) &> \pi_M(w', \tilde{q}', p^{BR}, I(p^{BR}, \tilde{q}')) \quad \forall t \notin T
\end{aligned}$$

Because deviations with a consistent interpretation can be seen as a self-fulfilling prophecy, the following additional requirement is imposed on equilibrium strategies:

Requirement 3. *There is no deviation with consistent interpretation in any equilibrium.*

Equilibria that fulfill this requirement will be referred to as 'Perfect Sequential Equilibria'.

Without the cheap-talk signal $\tilde{q}$, the model is a standard vertical relations model. It will be shown that while some information about the quality can be transmitted to consumers via the retail price, the manufacturer benefits from the introduction of an additional possibility of direct communication to the consumers. We will later see that the diverging interests of the vertical chain, that were already mentioned in the introduction, will allow M to credibly signal some of his private information to consumers.

In particular the manufacturer uses his private information strategically in a way that decreases the severity of the double marginalization problem occurring in a vertical chain where both firms possess market power.

4 Results

Instead of directly working with the full model, we will first look at a simplified version of it, namely one with one consumer of each type (i.e. $n_\ell = n_h = 1$), where one consumer's valuation of the good is independent of the quality, that is $\theta_\ell = 0$ ('the quality ignoring consumer') and the second consumer's quality sensitivity is set to one, i.e. $\theta_h = 1$ ('the quality aware consumer').

Consumers' valuations therefore are given as:

$$v_\ell = k \qquad \text{and} \qquad v_h(q) = k + q$$

While the assumed difference in valuations may seem rather extreme (and as will be shown in the next section, is not necessary for the result) it is easy to imagine such situations. For example, think about the good being a mobile phone and the let q measure the quality of the built-in camera. There certainly exist people who never use their mobile's camera and so they most probably don't care about its quality. At the same time many people are using the built-in camera and so are very well interested in the camera's specifications.

4.1 No Quality Signaling

Before dealing with the question if M can credibly transmit information about the quality of the good he produced to consumers, it is worthwhile to look at a situation where M does not possess this possibility.

This is crucial if we are interested to find out about M's incentives to signal information to consumers or not, and many of the arguments here can be used in the model with quality signaling with only minor changes

It is easy to see that in the given model, the retailer can not, on his own, inform consumers about the quality through the price. If, for a given wholesale price, the retailer would be charging different retail prices depending on the realization of the quality, the retailer would not act optimally. In this situation R would always be better off changing his strategy to one where he would always charge the higher retail price. Hence for a given wholesale price the retailer can only charge one retail price. However this does not rule out all possibilities that the retail price convey information about the quality. When setting his wholesale price, the manufacturer has to decide between setting a low wholesale price, trying to induce the retailer to serve all consumers, or setting a high wholesale price which in turn will induce the retailer to charge a high retail price only serving the quality aware consumer. As the following proposition shows separation is possible through the right choice of the wholesale pricing scheme. The retailer then passes on the information inherent in the wholesale price through his retail price.

To shorten notation, in everything that follows, the retailers' strategy will be represented by prices only, incorporating that he buys two goods from the manufacturer if his strategy is to set a retail price of p_h, he buys one good if $p = p_\ell$ and he buys no good if his strategy does not prescribe any retail price.

Proposition 1. *In the unique perfect sequential equilibrium in the simplified game without quality signaling, the type space Q is partitioned into* $Q_\ell = [0, \hat{q})$ *and* $Q_h = [\hat{q}, \overline{q}]$.[2]
The partitioning must be such that:

$$c(\hat{q}) = k - 3E(Q_h) \tag{1}$$

where $E(Q_i) := E(q|q \in Q_i)$. *Pricing schemes are given by:*

$$w = \begin{cases} w_\ell = k - E(Q_h) & \text{if} \quad q \in Q_\ell \\ w_h = k + E(Q_h) & \text{if} \quad q \in Q_h \end{cases}$$

$$p = \begin{cases} p_\ell = k & \text{if} \quad w \leq w_\ell \\ p_h = k + E(Q_h) & \text{if} \quad w \in (w_\ell, w_h] \end{cases}$$

The equilibrium is supported by beliefs of the form $\mu(p) = E(Q_h)\ \forall p > k$.

In equilibrium, optimal behavior implies that both consumers buy if the price is p_ℓ and only the high type does so when $p = p_h$.

It is easily verified that this indeed constitutes an equilibrium. Consumers act optimally since they either know that their valuation (weakly) exceeds the price (if $p = p_\ell$), or the price exactly equals the high type's expected valuation ($p = p_h$). Consumers' beliefs make it unprofitable for the retailer to set a price higher than p_h and for prices between p_ℓ and p_h the demand is constant so that such a price is always dominated by setting a price of p_h. Similarly all prices below p_ℓ are dominated by p_ℓ so that, given consumers' beliefs, the retailer will set a price of either p_ℓ or p_h. The specified off-equilibrium path beliefs yield a demand of zero for any price that is higher than p_h, which in turn implies that in this case the manufacturer can extract the full surplus by setting his wholesale price equal to the retail price $w = w_h = p_h$ making it impossible for the retailer to deviate. If the manufacturer wants to induce a retail price for which all consumers buy, the retailer has to be at least indifferent between setting this price p_ℓ and deviating to the high price p_h, which is exactly how w_ℓ is constructed. Finally, the manufacturer can now choose between setting a price of w_h and selling two units or selling one unit at a price of w_ℓ. As costs are increasing in the quality, the price-cost margin decreases for higher levels of q and at a quality $\hat{q}$ the manufacturer is indifferent between selling two products with a small margin or selling one product with a relatively higher margin. For qualities below $\hat{q}$, the small margin when selling two

[2]Note that this equilibrium requires $k > 3E(Q)$. Whenever this is not the case, the equation defining $\hat{q}$ has no solution and the resulting equilibrium is a pooling equilibrium which can be obtained from the proposition by setting $Q_\ell = \emptyset$ and $Q_h = Q$. Uniqueness of the equilibrium is up to the behavior of the two firms in the case that they are indifferent.

products is more than compensated by the number of goods sold, and for levels above $\hat{q}$, only one good is sold.

4.2 Quality Signaling

Before going through the model explicitly, it is worthwhile to think about the incentives a manufacturer might have to actually reveal some information about the quality to consumers.

By the structure of the demand, we can split up M's problem of maximizing his profit into two parts. M first calculates the maximal possible wholesale prices when selling to both consumers or to the quality caring consumer only, and in a second step, he then chooses his targeted consumers.

To build up intuition first think of an imaginary situation where M sends a quality signal $\tilde{q} \in [0, \overline{q}]$ and consumers literally believe this signal, that is $\mu(p, \tilde{q}) = \tilde{q}$.

Suppose first that M's goal is to sell to both consumers. He then knows that this can only be done at a retail price of at most k, which at the same time of course is the upper bound for the wholesale price in this situation. In the hypothetical situation where consumers take M's signal literally, M's best choice would be to send a signal of $\tilde{q} = 0$.

This would take away all possibilities from R to set a retail price higher than $p = k$, the price that leaves both consumers with an expected utility (given the signal) of zero. For a higher retail price no consumer would visit R.

The situation is quite different if M wants to sell to the quality caring consumer only. For any price that is higher than $p = k$ only one consumer demands the good and he only does so if the price is at most $k + \tilde{q}$.

As before M can extract the full surplus by setting his wholesale price equal to the retail price if only the high type consumer is to buy the good. Because the maximal retail price in this hypothetical situation is obtained for the highest possible signal, M would always signal $\tilde{q} = \overline{q}$ if he intends to sell to the quality caring consumer only.

Put differently, if M targets both consumers, his incentives for signaling information about the quality (for a given wholesale price), coincide with the consumers' incentives, namely keeping the retail price down. We can also interpret the signaling in this situation as means of M to decrease the severity of the well-known double marginalization problem. The lower the quality signal, the smaller the part of the profit that has to be granted to the retailer in order to keep him from deviating from the proposed strategy.

If, in contrast, the manufacturer targets only the quality caring consumer, the structure of demand implies that M's incentives are perfectly aligned with those of R, namely setting the retail price as high as possible. In this case, M can set his wholesale price equal to the retail price that will emerge. The retailer then has no choice but to follow the strategy of M, otherwise he won't make any sales (for a higher retail price) or make negative profits (for a lower retail price).[3]

Those observations and the intuition that because the gap in valuations between the two consumers increases with the produced quality so that selling to only the quality caring consumer is especially profitable for high realizations of the quality, give rise to the conjecture that the best the manufacturer could do is to find a cut-off quality level $\hat{q} \in [0, \overline{q}]$, so that M signals a low quality for qualities below this cut-off and a high quality for realizations of q above this level.

The following proposition shows that such a signaling scheme can indeed be part of an equilibrium of the simplified model, and in fact is used by the manufacturer in any perfect sequential equilibrium in this game.

Proposition 2. *In any perfect sequential equilibrium of the game with quality signaling, the type space Q is partitioned into $Q_\ell = [0, \hat{q})$ and $Q_h = [\hat{q}, \overline{q}]$.*
The partitioning must be such that:

$$c(\hat{q}) = k - 2E(Q_\ell) - E(Q_h) \tag{2}$$

with $E(Q_i) := E(q|q \in Q_i)$. Pricing and signaling schemes are given by:

$$(\tilde{q}, w) = \begin{cases} (\tilde{q}_\ell, w_\ell = k - E(Q_\ell)) & \text{if} \quad q \in Q_\ell \\ (\tilde{q}_h, w_h = k + E(Q_h)) & \text{if} \quad q \in Q_h \end{cases}$$

$$p = \begin{cases} p_\ell := k & \text{if} \quad w \leq w_\ell \\ p_h := k + E(Q_h) & \text{if} \quad w \in (w_\ell, w_h] \end{cases}$$

With beliefs of the following form:

$$E(Q_h) = \mu(p, \tilde{q}_h) \geq \mu(p, \tilde{q}) \geq \mu(p, \tilde{q}_\ell) = E(Q_\ell) \quad \forall p, \tilde{q} \notin \{\tilde{q}_\ell, \tilde{q}_h\}$$

In equilibrium, both consumers buy if the price is p_ℓ and only the high type does so when $p = p_h$.

The equilibrium is not unique, as the behavior of indifferent firms is arbitrary. Additionally, as stated in the proposition, the perfect sequential equilibrium allows for many off-equilibrium beliefs of the consumers.

Comparing the equilibria from Propositions 1 and 2, the wholesale pricing scheme directly shows that M's equilibrium profit is increased by sending signals to the consumers. Although the prices look similar, the difference is in the expected quality levels. Where there are the same

[3] This implies that the possibility of quality disclosure, i.e. truthfully revealing q at some positive costs would lead to a situation where the manufacturer discloses for low and high quality levels, but doesn't for intermediate qs. This contrasts the usual unraveling result, where only the highest types decide to disclose their type. A more detailed exploration of this topic is found in a companion paper (Conze, 2016).

(conditional) expectations for both wholesale prices in the game without direct information transmission, the formulas for w_ℓ and w_h contain different conditional expectations in an equilibrium with quality signaling and because it must be the case that $E(Q_\ell) \leq E(Q_h)$, the wholesale prices in informative equilibria are higher than those in non-informative ones.

In the separating equilibria without quality signaling, R could in principle always set a high price and make consumers believe that the good is of high quality. This made it necessary for the manufacturer to grant a relatively high share of the profits to the retailer in order to make him stick to setting a price that induces sales to all consumers. With the additional possibility of directly informing consumers, this is no longer true. By revealing that his good is of rather low quality, the deviation price that the retailer can set is decreased compared to a situation where M does not reveal such information. This decreased gain from a possible deviation makes it less costly for M to induce the retailer to set a price at which all consumers will buy, thereby increasing M's profits.

Theoretically, the messages $\tilde{q}_\ell$ and $\tilde{q}_h$ don't have to be specified, as long as they are understood. Nevertheless it might be worth to think about what they could refer to empirically.

Sending a high-quality signal could for example be interpreted as something along the lines of 'moon pricing' where in a model of reference dependent preferences, a manufacturer spoils consumers reference points by giving a high retail price recommendation (cf. Puppe and Rosenkranz, 2011 and Fabrizi et al., 2010). The situation here is similar if the message for a quality above the threshold is $\tilde{q}_h = \overline{q}$. Then the signaled quality is always (weakly) higher than the truly realized one, something which might also be observed in advertisings where quality (or other specifications) is often exaggerated in some way. The previous analysis nevertheless shows that such exaggerated advertisements can still contain valid information of interest to consumers.

4.3 General Model

The driving force behind the results of the last section is that the wholesale price the manufacturer can charge depends positively on the expected quality if the targeted consumer is the high type only and the situation is reversed if both types of consumers are targeted. While the first effect, that M can achieve a higher price the higher consumers expect their valuation to be, is intuitive, the effect that the wholesale price depends negatively on the expected quality if both consumers are targeted deserves some closer inspection.

In the simplified model, a higher expected quality translates into a higher willingness to pay of the high type. This in turn has two effects. On the one hand it increases the potential wholesale price for sales only to the high type. But on the other hand, it also increases the deviation incentives for the retailer whenever M intends to sell to both consumers. Since R's only meaningful deviation in such a situation is to sell to the high type only, this gets more profitable the higher this type's willingness to pay gets.

The goal of this section is to show how and under what restrictions the results of the simplified model, i.e. the possibility of credible quality signaling, can be transfered to the general model.

To briefly recap the general model: there are $n_\ell = 1$ low type and n_h high type consumers, whose sensitivity is given by θ_ℓ and θ_h respectively, with $\theta_\ell < \theta_h$. Consumers valuations are given as $v_i(q) = k + \theta_i q$.

For the proposition in this section we will also make the assumption that consumers trust the manufacturer's information, i.e. for any combinations $p, \tilde{q}$ and $p', \tilde{q}$ that are not on the equilibrium path, $\mu(p, \tilde{q}) = \mu(p', \tilde{q})$.

Proposition 3. *In any perfect sequential equilibrium of the game with quality signaling, where consumers differ sufficiently to fulfill $n_h\theta_h > m\theta_\ell$, the type space Q is partitioned into $Q_\ell = [0, \hat{q})$ and $Q_h = [\hat{q}, \overline{q}]$ with*

$$c(\hat{q}) = k - m(n_h\theta_h - m\theta_\ell)E(Q_\ell) - n_h\theta_h E(Q_h)$$

and price, signaling schemes and supporting beliefs are given by:

$$(\tilde{q}, w) = \begin{cases} (\tilde{q}_\ell, w_\ell(\tilde{q}_\ell) = (1+n_h)p_\ell(\tilde{q}_\ell) - n_h p_h(\tilde{q}_\ell) & \text{if} \quad q \in Q_\ell \\ (\tilde{q}_h, w_h(\tilde{q}_h) = p_h(\tilde{q}_h)) & \text{if} \quad q \in Q_h \end{cases}$$

$$p = \begin{cases} p_\ell(\tilde{q}_\ell) = k + \theta_\ell E(Q_\ell) & \text{if} \quad (\tilde{q}, w) = (\tilde{q}_\ell, w_\ell(\tilde{q}_\ell)) \\ p_h(\tilde{q}_h) = k + \theta_h E(Q_h) & \text{if} \quad (\tilde{q}, w) = (\tilde{q}_h, w_h(\tilde{q}_h)) \end{cases}$$

$$\mu(p, \tilde{q}_h) = E(Q_h) \geq \mu(p, q) \geq E(Q_\ell) = \mu(p, \tilde{q}_h) \quad \forall p, \tilde{q} \notin \{\tilde{q}_\ell, \tilde{q}_h\}$$

In equilibrium, both consumers buy if the price is p_ℓ and only the high type does so when $p = p_h$.

The effects that make credible signaling possible are the same as in the simplified model. M's setting of the wholesale price can be seen as the decision between setting a low wholesale price that induces (a retail price inducing) sales to all consumers and setting a high wholesale price that will only induce sales to the high type consumers. When M intends to induce sales to all consumers with a relatively low wholesale price, he has to make sure that the retailer has no incentive to deviate to another price at which only high type consumers purchase the good. In such a situation, a change in the expected quality of consumers which the manufacturer tries to influence with his signal, has two effects.

The first effect, which was already seen in the simplified model, is that an increased expected quality makes it more tempting for the retailer to only sell to high type consumers instead of to all consumers, which suggests trying to influence consumers' in a way that they decrease their expectation about the quality of the good.

An opposing effect, which before was ruled out by assuming that low types valuations are independent of the quality, arises since now all consumers' valuations are increasing for higher quality products. Hence a higher expected quality also raises the low types WTP, and thus the possible surplus that M can extract when low type consumers buy the good.

Whenever the first effect is stronger than the second, signaling a relatively low quality when inducing sales to all consumers is profitable, and the situation is similar to the simplified model. For this to be the case, consumers have to be sufficiently different as stated in the proposition.

5 Search Goods

The previous section dealt with the case of so-called experience goods. In many cases the assumption of such goods, that is the idea that consumers observe a good's quality only after actually consuming it may be too strict and modeling the good as a search good is more applicable.

A good is called a search good if some characteristics (here: quality) are unknown to consumers until they visit the point of sale. There they are able to observe the previously unknown characteristics of the good.

For this model to be different from one of perfect information, we have to introduce positive costs of visiting the retailer, and those shopping costs will be denoted by s. To rule out prohibitively large shopping costs it is assumed that they are smaller than the lowest possible valuation, that is $s < k$.

The change from experience to search goods greatly increases the necessary notation, so that this section will only use the simplified model from the previous section up to subsection 4.2. We thus again restrict the model to a situation where there are two consumers, one high-type and one low-type and the low-type's valuation for the good is independent of the realized quality, that is $m = 2, n_h = 1$ and $\theta_\ell = 0$ and $\theta_h = 1$.

The change of the nature of the good and the introduction of shopping costs are the only changes from the simplified model.

The following figure shows the adjusted timing for the model with a search good.

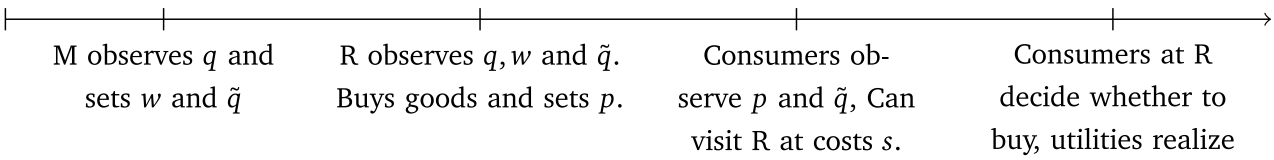

Figure 2: Model Timing for a Search Good

We will first look at a model without quality signaling of M, and then see if and how this can profitably be introduced.

The introduction of search costs makes it necessary for consumers to make two choices, first they have to decide whether to visit the retailer and if they did, they have to decide whether to buy the good or not. Clearly, consumers will decide to visit the retailer if they expect to gain a (weakly) positive utility from purchasing the good, i.e. consumer i will visit R whenever $k+\theta_i\mu(p) \geq p+s$. If a consumer decided to visit the retailer, his search costs are sunk and he will buy the good whenever his valuation is higher than the price charged, that is $v_i(q) \geq p$.

Those two conditions lead to the following equilibrium in the search goods game without quality signaling:

Proposition 4. *In the search goods model, there exists a partly separating equilibrium where the type space Q is partitioned into* $\{Q_N, ..., Q_1\}$ *as follows.*

The partitions Q_i *are constructed recursively, starting with the highest one:* $Q_1 = [q_1, \overline{q}]$ *such that* $q_1 = E(Q_1) - s$ *and* $Q_i = [q_i, q_{i-1}), i > 1$ *such that* $q_i = E(Q_i) - s$. *The lowest one is given as* $Q_N = [0, q_{N-1})$ *such that* $q_{N-1} > 0$ *and* $q_N = E(Q_N) - s \leq 0$.

$$w(q) = \begin{cases} w_\ell(Q_i) = k - E(Q_i) - s & \text{if} \quad q \in Q_i, c(q) \leq k - 3E(Q_i) - s \\ w_h(Q_i) = k + E(Q_i) - s & \text{if} \quad q \in Q_i, c(q) > k - 3E(Q_i) - s \end{cases}$$

$$p = \begin{cases} k - s & \text{if} \quad w = w_\ell(Q_i) \\ k + E(Q_i) - s & \text{if} \quad w = w_h(Q_i) \end{cases}$$

$$\mu(p) = E(Q_i) \qquad \text{if} \quad p - k + s \in Q_i$$

Sketch of Proof. The strategies closely resemble the strategies from the experience good model and the crucial observation here is that in each interval Q_i the situation essentially is the same as in the model of experience goods in that there are two possible retail and wholesale prices.

It is then easily checked that the proposed strategies indeed constitute and equilibrium.

For any proposed equilibrium price, the consumers are indifferent between visiting they retailer and not, and whenever they observe the quality at the retailer, they are always (weakly) better off by purchasing the good.

Given the beliefs, the wholesale pricing scheme is constructed in a way to keep the retailer from deviating, and given the retailer's and consumers' strategies, the manufacturer can not improve upon the proposed wholesale pricing scheme.

The reasoning behind the constructed intervals comes from observing that it can not happen in equilibrium that no consumer buys the good, since sales (and positive profits) are always achievable at a price of $p = k$. If it can not be that no consumer purchases the good, this means that at least one consumer must visit the retailer for any price on the equilibrium path, hence for all prices on the equilibrium path $E(v_h(q)|p) \geq p + c$ because whenever the low-type consumer buys the good, so does the high-type. This reasoning also implies that $v_h(q) > p$ for all prices on the equilibrium path. Putting those observations together with the fact that expectations have to be correct in equilibrium, the construction of the intervals assures that the retailer has no incentive to change the price, once consumers' search decision is sunk.

$\square$

With the observation that, by partitioning the space of possible qualities, the retail price now already delivers more information about the quality than was possible in the experience good model, the question is if M still is able to profitably send information to consumers.

With the observation that in each interval Q_i, the situation is almost identical to the experience goods setup, there should also be a possibility that the manufacturer informs the consumers.

More precisely, as the following proposition shows, M can indeed inform consumers about quality by dividing some interval Q_j into $\{Q_j^\ell, Q_j^h\}$, where a low wholesale price that induces sales to all consumers is set in Q_j^ℓ and a high wholesale price which equals the retail price is set in Q_j^h.

Proposition 5. *If* $k - s \in (2q_j + E(Q_j) + c(\overline{q}_i), 2E(Q_j) + q_{j-1} + c(\overline{q}_{i+1}))$ *for some interval* Q_j*, with intervals constructed according to the previous proposition, there is an equilibrium where the manufacturer reveals information to consumers via cheap-talk in this interval. Pricing schemes for all intervals different from* Q_j *remain as in the previous proposition. In* Q_j*:*

$$\tilde{q} = \begin{cases} \tilde{q}_\ell & \text{if} \quad q \leq q' \\ \tilde{q}_h & \text{else} \end{cases}$$

$$w(q) = \begin{cases} w_\ell(Q_j^\ell) := k - E(Q_j^\ell) - s & \text{if} \quad q \in Q_j, q \leq q' \\ w_h(Q_j^h) := k + E(Q_j^h) - s & \text{if} \quad q \in Q_j, q > q' \end{cases}$$

$$p = \begin{cases} k - c & \text{if} \quad w = w_\ell(Q_j^\ell) \\ w & \text{if} \quad w = w_h(Q_j^h) \end{cases}$$

$$\mu(p) = E(Q_j^f) \qquad \text{if} \quad p - k + s \in Q_j^f, f \in \{\ell, h\}$$

The threshold q' *is defined by:*

$$c(q') = k - s - 2E(Q_j^\ell) - E(Q_j^h) \tag{3}$$

Sketch of Proof. Since in the interval in question, the game is as in the previous section, the cheap-talk nature of the signal makes it necessary that M's profits are the same for for the quality realization q' for both signals sent and corresponding wholesale prices. As the costs are increasing, whenever a threshold exists, the interval Q_j is divided into $Q_j^\ell = \{q \in Q_j, q \leq q'\}$ and $Q_j^h = \{q \in Q_j, q > q'\}$. The threshold is similar to the one of Proposition 2, with additionally the shopping costs being taken into account. The condition from the beginning of the proposition ensures that the equation 3 defining the threshold has a solution.

If there is one interval, constructed according to the proposition such that this equation has a solution, M can credibly and profitably signal information to consumer even in the model of search goods.

□

While this proposition shows that signaling can also occur in a model of search goods it is clear that it's informativeness depends on the shopping costs s. The manufacturer can only reveal information that further divide one of the already existing intervals. And because smaller shopping costs induce a finer partition with smaller intervals, smaller shopping costs make the signal less informative. This can also be easily seen by considering two extreme cases. If the shopping costs go to zero, consumers can almost costlessly learn the quality of the good so that we approach a game of perfect information in which the price will either be $p = k + q - s$ or $p = k - s$. In this case information revelation of M is of no value. In the other extreme of very large shopping costs, the retail price can not partition the quality space, so that we are back in the situation of experience goods, where M's signaling is informative for a wide range of parameters.

This result also fits with the intuition, that the harder it is for consumers to get information (high s or experience goods) the more valuable is the information that the manufacturer can transmit. It can also be interpreted as confirming the claim of Nelson (1970) that informative advertising is more prevalent for experience than for search goods.

6 Conclusion

Conventional wisdom might suggest that manufacturers should always exaggerate the quality of their products, questioning the usefulness of such statements for consumers. Nevertheless announcements which can be interpreted as informations about quality are abundantly observed in reality.

This paper showed that incentives to do the exact opposite might arise as well in a situation where a manufacturer sells his goods indirectly through a retailer to heterogeneous consumers who are uninformed about the good's quality.

The goal when exaggerating a product's quality is clear, namely increasing consumer's willingness to pay, but the reasoning for downplaying the quality is less apparent. As this paper demonstrated, a vertical chain structure might lead to exactly those incentives.

If the manufacturer in such a situation has to decide what information about the quality of his good he wants to transmit to consumers, he has to take into account two potentially opposing effects.

On the one hand, and in line with conventional wisdom, leading consumers to believe that a good is of higher quality increases their willingness to pay, thereby increasing the possible profits the manufacturer can extract.

On the other hand, if consumers get increasingly heterogeneous for higher quality realizations, inducing consumers to believe the produced good is of high quality increases the deviation incentives for the retailer for prices where the demand is elastic, so that it can become optimal to induce low quality expectations with consumers.

Phrased differently, the manufacturer can use strategic information transmission to consumers as means to control the retailers freedom in price setting, which directly influences his deviation incentives. If the retailer has less incentives to deviate, the manufacturer can charge higher wholesale prices for the same retail price thereby increasing his profits.

While most empirical examples that come to mind when thinking about information transmission from manufacturers to consumers seem to follow the theme of exaggeration there are some examples where informations sent from manufacturers could lead consumers to expect a good of a lower quality.

One of those examples are retail price recommendations. A (relatively) low retail price recommendation probably leads consumers to expect a good to be of rather low quality. Although the face value of such RPR might not provide much information (just as the signals in the model with cheap talk are not pinned down), the comparison of different RPRs gives consumers an orientation about what quality to expect. Besides RPRs, many manufacturers sell their goods under the name of different brands and those brands often carry specific projections about quality. The Taiwanese producer of computer parts 'Asustek' for example, founded a sub-company concentrating on the low-price segment called 'ASRock' and many car manufacturers (like VW) produce cars of a variety of brands.

Appendix A Proofs

This section provides the proofs of the theorems in the main text, which are restated here for convenience.

Proposition 1. *In the unique perfect sequential equilibrium in the simplified game without quality signaling, the type space Q is partitioned into $Q_\ell = [0, \hat{q})$ and $Q_h = [\hat{q}, \overline{q}]$.*[4]
The partitioning must be such that:

$$c(\hat{q}) = k - 3E(Q_h) \tag{1}$$

where $E(Q_i) := E(q|q \in Q_i)$. Pricing schemes are given by:

$$w = \begin{cases} w_\ell = k - E(Q_h) & \text{if} \quad q \in Q_\ell \\ w_h = k + E(Q_h) & \text{if} \quad q \in Q_h \end{cases}$$

$$p = \begin{cases} p_\ell = k & \text{if} \quad w \leq w_\ell \\ p_h = k + E(Q_h) & \text{if} \quad w \in (w_\ell, w_h] \end{cases}$$

The equilibrium is supported by beliefs of the form $\mu(p) = E(Q_h)\ \forall p > k$.

In order to prove this proposition, we will first establish some properties that any equilibrium in this game must have, then show what equilibria exist and in the last step argue that other equilibria are not sequentially perfect.

Proof.
Step 1: R's optimal strategy can at most be binary.
Fix a set of beliefs μ and let the sets $P_i, i \in \{\ell, h\}$ denote all prices that induce sales to a consumer of type i, that is: $P_i = \{p : p \leq k + \theta_i \mu(p)\}$. The sets P_i are non-empty since $p = k$ must be part of both. Let p_i be the highest price that induces sales to consumers of type i, $p_i = \max P_i$. In the simplified model, $\theta_\ell = 0$ so that $p_\ell = k$. Because any price that induces sales to the low-types will also induce sales to the high type, $P_\ell \subseteq P_h$ and $p_h \geq p_\ell$. The retailer's optimal strategy for any $w \leq p_h$ is

$$p^*(w) = \arg \max_{p \in \{p_\ell, p_h\}} \pi_R(p, w).$$

Step 2: If $p^*(w) = p_h$ for some w, by setting $w = w_h := p_h$, M can extract the whole surplus when sales are made to the high type consumer only. With $w = p_h$, the retailer is indifferent between

[4] Note that this equilibrium requires $k > 3E(Q)$. Whenever this is not the case, the equation defining $\hat{q}$ has no solution and the resulting equilibrium is a pooling equilibrium which can be obtained from the proposition by setting $Q_\ell = \emptyset$ and $Q_h = Q$. Uniqueness of the equilibrium is up to the behavior of the two firms in the case that they are indifferent.

buying the good from M and setting $p = p_h$ or not buying the good, and so by assumption he buys the good and sets $p = p_h$.

Step 3: M can induce a retail price of $p = k$.

R will set a price of $p = p_\ell = k$ if this yields weakly higher profits than the best deviation. As Step 1 shows that the only deviation is to set a price of p_h this means:

$$2(p_\ell - w) \geq p_h - w$$
$$w \leq w_\ell := 2k - p_h$$

Step 4: M's optimal strategy partitions Q into Q_ℓ and Q_h.

By setting $w = w_h$, M can always make positive profits, implying it can't be the case that no sales are made for some realization of q. Step 1 showed that R's strategy is at most binary and Steps 2 and 3 showed that retail prices p_ℓ and p_h can be induced by wholesales prices w_ℓ and w_h respectively, since w_i is the maximal wholesale price that will induce $p_i, i \in \{\ell, h\}$, M's optimal strategy $w^*(q)$ will only contain those two wholesale prices. Together this implies that M partitions Q into Q_ℓ and Q_h for which wholesale prices are w_ℓ and w_h respectively. Thus $Q_\ell \cup Q_h = Q = [0, \overline{q}]$ and because we concentrate on pure strategies $Q_\ell \cap Q_h = \emptyset$, hence:

$$w^*(q) = \arg \max_{w \in \{w_\ell, w_h\}} \pi_M(w)$$

Step 5: Beliefs have to be correct in equilibrium, i.e. $\mu(p_\ell) = E(Q_\ell) := E(q|q \in Q_\ell)$ and $\mu(p_h) = E(Q_h) := E(q|q \in Q_h)$ and thus $w_h = p_h = k + E(Q_h)$ and $w_\ell = k - E(Q_h)$.

In an equilibrium, R will set prices $p_\ell = k$ and $p_h = k + \mu(p_h) = k + E(Q_h)$, and by Step 2, $w_h = p_h$ and $w_\ell = 2k - p_h = k - E(Q_h)$.

Step 6: M prefers to induce a price of p_ℓ by charging a wholesale price w_ℓ if $\pi_M(w_\ell) > \pi_M(w_h)$, that is whenever

$$2(k - E(Q_h) - c(q)) > k + E(Q_h) - c(q)$$
$$c(q) < k - 3E(Q_h).$$

If the converse is true, M is better off by setting w_h. For any fixed partition Q_ℓ, Q_h, if the condition holds for some q it must also hold for any smaller q, since the left-hand side is increasing in q. With these observations, there is a unique quality level $\hat{q}$ for which M is indifferent and as stated in the proposition, $\hat{q}$ is given by $c(\hat{q}) = k - 3E(Q_h)$. The partition must thus be of the form $\{Q_\ell = [0, \hat{q}), Q_h = [\hat{q}, \overline{q}]\}$.

Step 7: The previous steps established the existence of a separating equilibrium and showed that no other separating equilibrium could possibly exist. It remains to show that pooling equilibria are either non-existent or ruled out by the refinement.

Two pooling equilibria are generally possible, depending on which consumers are served.

Start with a candidate for a pooling equilibrium where only the high-type consumer is served, thus $p = w = k + \mu(p) = k + E(Q)$. Lower prices can not be sustained since the retailer would always have an incentive to deviate and the beliefs are on the equilibrium path and thus have to be correct. In this situation, M could induce a price of $p' = k$ where both consumers are served by choosing $w' = k - E(Q)$ which he prefers to do whenever:

$$\begin{aligned}
\pi_M(w') = 2(w' - c(q)) &> w - c(q) = \pi_M(w) \\
2(k - E(q) - c(q)) &> k + E(q) - c(q) \\
c(q) &< k - 3E(q)
\end{aligned}$$

which, by assumption is true for small realizations of q. There can thus be no pooling equilibrium where only the high-type consumer is served.

It remains to show that a candidate pooling equilibrium where both consumers are served is not sequentially perfect. In such an equilibrium the previous steps imply that the wholesale price would have to equal $w = k - \mu(p_h)$ where again $p_h = \max p$ s.t. $p \leq k + \mu(p)$. Instead of pooling on the wholesale price w, M could always induce a price of p_h by setting $w = w_h = p_h$, so for the desired pooling equilibrium it must be the case that

$$2(w - c(q)) = 2(k - \mu(p_h) - c(q)) > k + \mu(p_h) - c(q) = w_h - c(q)$$

for all realizations of q, in particular also for $\overline{q}$:

$$c(\overline{q}) < k - 3\mu(p_h)$$

From the assumption on the cost function, we know that $c(\overline{q}) > k - 3E(q)$ so that the above condition is violated whenever $\mu(p_h) > E(q)$.

Therefore suppose $\mu(p_h) < E(q)$ for the remainder. By construction w makes the retailer indifferent between setting a price of $p = k$ and $p_h = k + \mu(p_h)$. If $\mu(p_h) < E(q)$ the retailer would be better off by setting a price of $p' = k + E(q)$ irrespectively of the realized quality. If consumers were to observe a deviation price of p', they would have to reason that this deviation price is profitable for the retailer for all realized qualities so that they would attribute this deviation price to the whole type space. With the interpretation $I(p') = Q$ which leads to beliefs of $\mu(p') = E(q)$, the retailer would indeed deviate to this price for all q, so that p' and $I(p') = Q$ constitute a deviation with a consistent interpretation.

This shows that there is no sequentially perfect pooling equilibrium in this game.

□

Proposition 2. *In any perfect sequential equilibrium of the game with quality signaling, the type space Q is partitioned into $Q_\ell = [0, \hat{q})$ and $Q_h = [\hat{q}, \overline{q}]$.*
The partitioning must be such that:

$$c(\hat{q}) = k - 2E(Q_\ell) - E(Q_h) \tag{2}$$

with $E(Q_i) := E(q|q \in Q_i)$. Pricing and signaling schemes are given by:

$$(\tilde{q}, w) = \begin{cases} (\tilde{q}_\ell, w_\ell = k - E(Q_\ell)) & \text{if} \quad q \in Q_\ell \\ (\tilde{q}_h, w_h = k + E(Q_h)) & \text{if} \quad q \in Q_h \end{cases}$$

$$p = \begin{cases} p_\ell := k & \text{if} \quad w \leq w_\ell \\ p_h := k + E(Q_h) & \text{if} \quad w \in (w_\ell, w_h] \end{cases}$$

With beliefs of the following form:

$$E(Q_h) = \mu(p, \tilde{q}_h) \geq \mu(p, \tilde{q}) \geq \mu(p, \tilde{q}_\ell) = E(Q_\ell) \quad \forall p, \tilde{q} \notin \{\tilde{q}_\ell, \tilde{q}_h\}$$

Proof.
Step 1: For a given quality signal $\tilde{q}$, R's optimal strategy can at most be binary.
Fix a set of beliefs μ and a signal $\tilde{q}$ and let the sets $P_i(\tilde{q}), i \in \{\ell, h\}$ denote all prices that induce sales to a consumer of type i, given signal $\tilde{q}$, that is: $P_i(\tilde{q}) = \{p : p \leq k + \theta_i \mu(p, \tilde{q})\}$. The sets P_i are non-empty since $p = k$ must be part of both. Let $p_i(\tilde{q})$ be the highest price that induces sales to consumers of type i, $p_i(\tilde{q}) = \max P_i(\tilde{q})$. In the simplified model, $\theta_\ell = 0$ so that $p_\ell = k$, independent of $\tilde{q}$. Because any price that induces sales to the low-types will also induce sales to the high type, $P_\ell \subseteq P_h(\tilde{q})$ and $p_h(\tilde{q}) \geq p_\ell$ for any signal $\tilde{q}$. The retailer's optimal strategy for any $w \leq p_h(\tilde{q})$ is

$$p^*(w, \tilde{q}) = \arg \max_{p \in \{p_\ell, p_h(\tilde{q})\}} \pi_R(p, w)$$

Step 2: Given R's optimal strategy and for signal $\tilde{q}$, by setting $w = w_h(\tilde{q}) := p_h(\tilde{q})$, M can extract the whole surplus when sales are made to the high type consumer only.
Step 3: M can induce a retail price of $p = k$.
R will set a price of $p = p_\ell = k$ if this yields weakly higher profits than the best deviation. As Step 1 shows that the only deviation is to set a price of $p_h(\tilde{q})$, this means:

$$\begin{aligned} 2(p_\ell - w) &\geq p_h(\tilde{q}) - w \\ w &\leq w_\ell(\tilde{q}) := 2k - p_h(\tilde{q}) \end{aligned}$$

Step 4: M's optimal strategy partitions Q into Q_ℓ and Q_h.
By setting $w = w_h(\tilde{q})$ for some signal $\tilde{q}$, M can always make positive profits, implying it can't be the case that no sales are made for some realization of q.

Steps 2 and 3 showed that retail prices p_ℓ and $p_h(\tilde{q})$ can be induced by wholesales prices $w_\ell(\tilde{q})$ and $w_h(\tilde{q})$. Hence, if M is going to send a signal $\tilde{q}$ to consumers, it can only be the case that one signal is sent per corresponding wholesale price, i.e.:

$$w^*(q) = \arg \max_{w \in \{w_\ell(\tilde{q}_\ell), w_h(\tilde{q}_h)\}} \pi_M(w)$$

with

$$\tilde{q}_i = \arg\max_{\tilde{q}} w_i(\tilde{q}) \quad i \in \{\ell, h\}$$

Step 5: Beliefs on the equilibrium path have to be correct, i.e. $\mu(p_\ell, \tilde{q}_\ell) = E(Q_\ell) := E(q|q \in Q_\ell)$ and $\mu(p_h(\tilde{q}_h), \tilde{q}_h) = E(Q_h) := E(q|q \in Q_h)$ and thus $w_h = p_h(\tilde{q}_h) = k + E(Q_h)$.

Step 6: By the same arguments as in the previous proposition there is a cut-off $\hat{q}$ that equalizes M's profits for both combinations of signals and wholesale prices, hence $\hat{q}$ is defined through:

$$\begin{aligned}2(w_\ell(\tilde{q}_\ell)-c(\hat{q})) &= w_h(\tilde{q}_h)-c(\hat{q})\\ 2(k-\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell)-c(\hat{q})) &= k+\mu(p_h(\tilde{q}_h),\tilde{q}_h)-c(\hat{q}) = k+E(Q_h)-c(\hat{q})\\ c(\hat{q}) &= k-2\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell)-E(Q_h)\end{aligned}$$

Step 7: The combination of $(\tilde{q}_\ell, p_h(\tilde{q}_\ell))$ is off the equilibrium path and thus $\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell)$ is not pinned down by the requirements of a weak perfect Bayesian equilibrium. But, as this step will show, perfect sequentiality of the equilibrium requires $\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell) = \mu(p_\ell(\tilde{q}_\ell),\tilde{q}_\ell) = E(Q_\ell)$. Let $\mu_Q(\tilde{q}', p')$ be the expected quality after an observed deviation to $(\tilde{q}', p')$ formed according to the interpretation $I(\tilde{q}', p') = Q$.

Take a candidate equilibrium with price, wholesale and signaling scheme constructed according to the previous steps and the partitioning of Q_ℓ and Q_h according to equation (2). Remember that by construction of $\tilde{q}_\ell$ and p_h it must be the case that $\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell) \geq E(Q_\ell)$ and assume $\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell) > E(Q_\ell)$ in the candidate equilibrium.

Given that the manufacturer can send a different signal to consumers the task is to find a deviation that involves a different wholesale price and a different signal with a consistent interpretation. Consider a deviation to $w', \tilde{q}'$ with $w_h > w' > w_\ell$ so that all consumers would purchase the good. With consumers estimated quality being $\mu_Q(\tilde{q}', p)$ after observing the deviation signal, M can induce a price of $p = k$ by setting $w' = k - \mu_Q(\tilde{q}', p)$. Assuming that $w' > w_\ell$ M would prefer this deviation for all $q \in Q_\ell$. For $q \in Q_h$ M would prefer w' over w_h if

$$\begin{aligned}2(w'-c(q)) &> k+E(Q_h)-c(q)\\ c(q) &> k-2\mu_Q(\tilde{q}', p)-E(Q_h)\end{aligned}$$

the interpretation $\mu_{Q'}(\tilde{q}', p) = E(Q')$ where $Q' = [0, q']$ is consistent if

$$\begin{aligned}c(q) &> k-2E(Q')-E(Q_h) \quad \forall q \in Q'\\ c(q) &< k-2E(Q')-E(Q_h) \quad \forall q \notin Q'\end{aligned}$$

so that a 'deviation threshold' q' can be constructed as $c(q') = k - 2E(Q') - E(Q_h)$. It remains to check that indeed $w' > w_\ell$, i.e. $E(Q') < \mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell)$. From combining the condition defining the 'original' threshold $\hat{q}$, $c(\hat{q}) = k - 2\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell) - E(Q_h)$ with the equation defining the 'deviation threshold' we get:

$$c(q') + 2E(Q') = c(\hat{q}) + 2\mu(p_h(\tilde{q}_\ell),\tilde{q}_\ell) > c(\hat{q}) + 2E(Q_\ell)$$

which implies $q' > \hat{q}$ and thus

$$c(q') + 2E(Q') = c(\hat{q}) + 2\mu(p_h(\tilde{q}_\ell), \tilde{q}_\ell)$$

means

$$E(Q') < \mu(p_h(\tilde{q}_\ell), \tilde{q}_\ell)$$

The above showed that if $\mu(p_h(\tilde{q}_\ell), \tilde{q}_\ell) > E(Q_\ell)$ in a weak perfect Bayesian equilibrium, this equilibrium is ruled out by perfect sequentiality. Intuitively, in an equilibrium that is not sequentially perfect, consumers are giving the retailer too much 'power' in forming their beliefs although a deviation of only the retailer can not contain any additional information about the quality, so that beliefs should not change in response to only price changes. This is exactly what the refinement of perfect sequentiality formalizes.

Step 8: The same reasoning of the last step in the previous proposition can be used to show that no pooling equilibrium can be sequentially perfect.

□

Proposition 3. *In any perfect sequential equilibrium of the game with quality signaling, where consumers differ sufficiently to fulfill $n_h\theta_h > m\theta_\ell$, the type space Q is partitioned into $Q_\ell = [0, \hat{q})$ and $Q_h = [\hat{q}, \overline{q}]$ with*

$$c(\hat{q}) = k - m(n_h\theta_h - m\theta_\ell)E(Q_\ell) - n_h\theta_h E(Q_h)$$

and price, signaling schemes and supporting beliefs are given by:

$$(\tilde{q}, w) = \begin{cases} (\tilde{q}_\ell, w_\ell(\tilde{q}_\ell) = (1+n_h)p_\ell(\tilde{q}_\ell) - n_h p_h(\tilde{q}_\ell) & \text{if} \quad q \in Q_\ell \\ (\tilde{q}_h, w_h(\tilde{q}_h) = p_h(\tilde{q}_h)) & \text{if} \quad q \in Q_h \end{cases}$$

$$p = \begin{cases} p_\ell(\tilde{q}_\ell) = k + \theta_\ell E(Q_\ell) & \text{if} \quad (\tilde{q}, w) = (\tilde{q}_\ell, w_\ell(\tilde{q}_\ell)) \\ p_h(\tilde{q}_h) = k + \theta_h E(Q_h) & \text{if} \quad (\tilde{q}, w) = (\tilde{q}_h, w_h(\tilde{q}_h)) \end{cases}$$

$$\mu(p, \tilde{q}_h) = E(Q_h) \geq \mu(p, q) \geq E(Q_\ell) = \mu(p, \tilde{q}_h) \quad \forall p, \tilde{q} \notin \{\tilde{q}_\ell, \tilde{q}_h\}$$

The proof is almost the same as in the previous proposition, with the biggest difference being that the retail price that induces sales to all consumers now also depends on consumers' beliefs.

Proof.
Step 1: For a given quality signal $\tilde{q}$, R's optimal strategy can at most be binary.
Fix a set of beliefs μ and a signal $\tilde{q}$ and let the sets $P_i(\tilde{q}), i \in \{\ell, h\}$ denote all prices that induce sales to a consumer of type i, given signal $\tilde{q}$, that is: $P_i(\tilde{q}) = \{p : p \leq k + \theta_i\mu(p, \tilde{q})\}$. The sets P_i

are non-empty since $p = k$ must be part of both. And $P_\ell \subseteq P_h(\tilde{q})$ and $p_h(\tilde{q}) \geq p_\ell(\tilde{q})$ for any signal $\tilde{q}$.

The retailer's optimal strategy for any $w \leq p_h(\tilde{q})$ is

$$p^*(w, \tilde{q}) = \arg \max_{p \in \{p_\ell(\tilde{q}), p_h(\tilde{q})\}} \pi_R(p, w)$$

Step 2: If for a fixed $\tilde{q}$, $p = p_h(\tilde{q})$, M can set $w = w_h(\tilde{q}) = p_h(\tilde{q})$.
Step 3: R will set $p = p_\ell(\tilde{q})$ if:

$$\begin{aligned}(1 + n_h)(p_\ell(\tilde{q}) - w) &\geq n_h(p_h(\tilde{q}) - w)\\ w &\leq w_\ell(\tilde{q}) := p_\ell(\tilde{q}) - n_h(p_h(\tilde{q}) - p_\ell(\tilde{q}))\end{aligned}$$

As before prices will be such that consumers' expected utility equals zero, i.e. $p_i(\tilde{q}) = k + \mu(\tilde{q}, p_i(\tilde{q}))$.

Hence,

$$\begin{aligned}w_\ell(\tilde{q}) &= k + m\theta_\ell \mu(\tilde{q}, p_\ell(\tilde{q})) - n_h \theta_h \mu(\tilde{q}, p_h(\tilde{q}))\\ w_h(\tilde{q}) &= k + \theta_h \mu(\tilde{q}, p_h(\tilde{q}))\end{aligned}$$

$w_h(\tilde{q})$ is increasing in consumers expectation, thus, whenever $w_\ell(\tilde{q})$ is decreasing in μ, M will optimally choose different signals to accompany each wholesale price. We will for now just assume that this is the case and come back to the conditions on the parameters that guarantee this later on.

M's optimal strategy then is.

$$w^*(q) = \arg \max_{w \in \{w_\ell(\tilde{q}_\ell), w_h(\tilde{q}_h)\}} \pi_M(w)$$

with

$$\tilde{q}_i = \arg \max_{\tilde{q}} w_i(\tilde{q}) \quad i \in \{\ell, h\}$$

Step 4: M's optimal strategy partitions Q into Q_ℓ and Q_h.
By setting $w = w_h(\tilde{q})$ for some signal $\tilde{q}$, M can always make positive profits, implying it can't be the case that no sales are made for some realization of q.
Step 5: As in the two previous propositions, a unique threshold $\hat{q}$ that equalizes the profits from both combinations of signals and wholesale prices, hence $\hat{q}$ is defined through:

$$
\begin{aligned}
\pi_M(\tilde{q}_\ell, w_\ell(\tilde{q}_\ell)) &= \pi_M(\tilde{q}_h, w_h(\tilde{q}_h)) \\
m w_\ell(\tilde{q}_\ell) &= n_h w_h(\tilde{q}_h) \\
c(\hat{q}) &= k + m^2\theta_\ell \mu(\tilde{q}_\ell, p_\ell(\tilde{q}_\ell)) - n_h\theta_h[m\mu(\tilde{q}_\ell, p_h(\tilde{q}_\ell)) + \mu(\tilde{q}_h, p_h(\tilde{q}_h))]
\end{aligned}
$$

Step 6: Beliefs on the equilibrium path have to be correct, i.e. $\mu(p_\ell, \tilde{q}_\ell) = E(Q_\ell)$ and $\mu(p_h(\tilde{q}_h), \tilde{q}_h) = E(Q_h)$, so that

$$
\begin{aligned}
w_\ell(\tilde{q}_\ell) &= k + m\theta_\ell E(Q_\ell) - n_h\theta_h\mu(\tilde{q}, p_h(\tilde{q})) \\
w_h(\tilde{q}_h) &= k + \theta_h E(Q_h)
\end{aligned}
$$

Step 7: As in Proposition 2, the on-equilibrium wholesale price $w_\ell(\tilde{q}_\ell)$ depends on an off-equilibrium belief, namely on $\mu(\tilde{q}, p_h(\tilde{q}))$. The same Arguments from Step 7 of Proposition 2 imply that in any perfectly sequential equilibrium $\mu(\tilde{q}, p_h(\tilde{q})) = E(Q_\ell)$ so that again a deviation by only the retailer can not contain information. Thus $w_\ell(\tilde{q}_\ell) = k + m\theta_\ell E(Q_\ell) - n_h\theta_h\mu(\tilde{q}, p_h(\tilde{q})) = k + E(Q_\ell)(m\theta_\ell - n_h\theta_h)$ is decreasing in the expected quality, as wanted, whenever $n_h\theta_h > m\theta_\ell$. The cut-off can then be rewritten as

$$
c(\hat{q}) = k - (n_h\theta_h - m\theta_\ell)mE(Q_\ell) - n_h\theta_h E(Q_h)
$$

□

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